Are the $X(4160)$ and $X(3915)$ charmonium states?

You-chang Yang[1,2], Zurong Xia[1], Jialun Ping[1*]
[1]*Department of Physics, Nanjing Normal University, Nanjing 210097, P. R. China*
[2]*Department of Physics, Zunyi Normal College, Zunyi 563002, P. R. China*

Inspired by the newly observed $X(4160)$ and $X(3915)$ states, we analyze the mass spectrum of these states in different quark models and calculate their strong decay widths by the $^3P_0$ model. According to the mass spectrum of charmonium states predicted by the potential model, the states $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$, $\eta_c(4^1S_0)$ all can be candidates for the $X(4160)$. However, only the decay width of the state $\eta_{c2}(2^1D_2)$ in our calculation is in good agreement with the data reported by Belle and the decay of $\eta_{c2}(2^1D_2) \to D\bar{D}$, which is not seen in experiment, is also forbidden. Therefore, it is reasonable to interpret the charmonium state $\eta_{c2}(2^1D_2)$ as the state $X(4160)$. For the state $X(3915)$, although the mass of $\chi_0(2^3P_0)$ is compatible with the experimental value, the calculated strong decay width is much larger than experimental data. Hence, the assignment of $X(3915)$ to charmonium state $\chi_0(2^3P_0)$ is disfavored in our calculation.

PACS numbers: 14.40.Pq, 13.25.Gv, 12.38.Lg

I. INTRODUCTION

Many new charmonium like states, the so-called XYZ mesons, have been reported by Belle and BaBar collaborations in recent years. Some of these states can be understood as conventional mesons that are comprised of only pure $c\bar{c}$ quark pair. However, most of the XYZ states do not match well the mass spectrum of $c\bar{c}$ predicted by the QCD-motivated potential models. By considering the effects of virtual mesons loop [1, 2, 3, 4] and color screening [6], the masses of some excited charmonium states are smaller than it calculated by conventional quark model. Therefore, some XYZ states [2] may be still compatible with the mass spectrum of charmonium. However, the state $X(3872)$ [2, 4, 5] is probably the most robust of all the charmonium like objects.

Last year, Belle collaborations reported a new charmonium like state, the $X(4160)$ [7], in the processes $e^+e^- \to J/\psi D^{(*)}\bar{D}^{(*)}$ with a significance of 5.1σ. It has the mass $M = 4156^{+25}_{-20} \pm 15$ MeV, and width $\Gamma = 139^{+111}_{-61} \pm 21$ MeV. Based on the the processes $e^+e^- \to J/\psi D\bar{D}$, $e^+e^- \to J/\psi D^*\bar{D}$, and $e^+e^- \to J/\psi D^*\bar{D}^*$, The upper limits of the branch ratios of $X(4160)$ are given as,

$$\mathcal{B}_{D\bar{D}}(X(4160))/\mathcal{B}_{D^*\bar{D}^*}(X(4160)) < 0.09,$$
$$\mathcal{B}_{D^*\bar{D}}(X(4160))/\mathcal{B}_{D^*\bar{D}^*}(X(4160)) < 0.22.$$

The $X(4160)$ has possible charge parity $C = +$ mostly, since the photon γ and J/ψ have $J^{PC} = 1^{--}$, and $e^+e^- \to \gamma \to J/\psi X(4160)$ is a main process. Hence the $X(4160)$ can have $J^{PC} = 0^{-+}, 0^{++}, 1^{-+}, 2^{-+}, 1^{++}, 2^{++}, \ldots$. In Ref.[12], Chao discussed the possible interpretation of the $X(4160)$ in view of production rate in $e^+e^- \to J/\psi X(4160)$. He believes that the charmonium states 4^1S_0, 3^3P_0 may be assigned to the state $X(4160)$ by analogy with the cross section of $e^+e^- \to J/\psi\eta_c(1S)(\eta_c(2S)\chi_{c0}(1P))$, while the 2^1D_2 [14] can not be rule out. According to the mass spectrum [6] predicted by the potential model with color screening, Li and Chao also give some arguments about the $\chi_0(3^3P_0)$ as an assignment for the $X(4160)$.

Using the vector-vector interaction within the framework of the hidden gauge formalism, Molina and Oset [15] suggested that the $X(4160)$ is a molecular state of $D_s^*\bar{D}_s^*$ with $J^{PC} = 2^{++}$.

Very recently, Refs.[8, 9, 10, 11] reported the newest charmonium like state, the $X(3915)$, which is observed by Belle in $\gamma\gamma \to \omega J/\psi$ with a statistical significance of 7.5σ. It has the mass and width

$$M = 3914 \pm 4 \pm 2 \text{ MeV}, \quad \Gamma = 28 \pm 12^{+2}_{-8} \text{ MeV}.$$

Belle collaborations determine the $X(3915)$ production rate $\Gamma_{\gamma\gamma}(X(3915))\ \mathcal{B}(X(3915) \to \omega J/\psi) = 69 \pm 16^{+7}_{-18}$ eV and $\Gamma_{\gamma\gamma}(X(3915))\ \mathcal{B}(X(3915) \to \omega J/\psi) = 21 \pm 4^{+2}_{-5}$ eV for $J^P = 0^+$ or 2^+, respectively. Because the partial width of this state to $\gamma\gamma$ or $\omega J/\psi$ is too large, it is very unlikely to be a charmonium state analyzed by Yuan [9].

The $X(3915)$ also has the charge parity $C = +$, because it is observed in the process of $\gamma\gamma \to \omega J/\psi$. In Ref.[21], Liu *et al.* argued that the $\chi_0(2^3P_0)$ can be assigned to the $X(3915)$ if taking $R = 1.8 \sim 1.85$ GeV^{-1} in the SHO (the simple harmonic oscillator wave functions).

Up to now, the interpretation of the $X(4160)$ and $X(3915)$ is still unclear. The states $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$ listed in Table I all can be interpreted as the $X(4160)$ just on mass level. Which charmonium state is an assignment for the $X(4160)$? One can answer this question in different ways. We study the $X(4160)$ and $X(3915)$ via strong decay by the $^3P_0$ model [16, 17, 18, 19] in this work. In following discussion, we take the $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$, $\eta_c(4^1S_0)$ and $\chi_0(2^3P_0)$ as candidates of the $X(4160)$ and $X(3915)$, respectively.

The paper is organized as follows. In the next section we take a review of the $^3P_0$ model. Sect. III devotes

*Electronic address: jlping@njnu.edu.cn

TABLE I: Theoretical mass spectrum of the charmonium candidates for the $X(4160)$ and $X(3915)$. The mass are in units of MeV. The results are taken from Ref.[6] with color screening potential model, and Ref.[13] including Nonrelativistic potential and Godfrey-Isgur relativized potential model.

State J^{PC}	$\chi_0(2^3P_0)$ 0^{++}	$\eta_c(4^1S_0)$ 0^{-+}	$\chi_0(3^3P_0)$ 0^{++}	$\chi_1(3^3P_1)$ 1^{++}	$\eta_{c2}(2^1D_2)$ 2^{-+}
Ref.[6] SCR	3842	4250	4131	4178	4099
Ref.[13] NR	3852	4384	4202	4271	4158
Ref.[13] GI	3916	4425	4292	4317	4208

to discuss the possible strong decay channels and gives the corresponding amplitudes of the candidates for the $X(4160)$ and $X(3915)$. In Sect. IV we present and analyze the results obtained by the $^3P_0$ model. Finally, the summary of the present work is given in the last section.

II. A REVIEW OF THE $^3P_0$ MODEL OF MESON DECAY

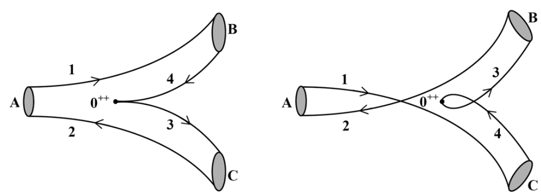

Fig.1 The two possible diagrams contributing to $A \to B + C$ in the $^3P_0$ model.

The $^3P_0$ decay model, also known as the Quark-Pair Creation model (QPC), was originally introduced by Micu[16] and further developed by Le Yaouanc, Ackleh, Roberts *et al.*[17, 18, 19]. It is applicable to OZI (Okubo, Zweig and Iizuka) rule allowed strong decays of a hadron into two other hadrons, which are expected to be the dominant decay modes of a hadron. Due to the $^3P_0$ model gives a good description of many observed partial widths of the hadrons, it has been widely used to evaluate the strong decays of mesons and baryons composed of u, d, s, c, b quarks [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. The $^3P_0$ model of strong decays assumes that quark-antiquark pair are created with vacuum quantum number $J^{PC} = 0^{++}$[16]. The diagrams of all possible decay process $A \to B+C$ of meson are shown in Fig.1. In many cases only one of them contributes to the strong decay of meson.

The transition operator of this model takes

$$T = -3\,\gamma \sum_m \langle 1m1-m|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3+\mathbf{p}_4) \times \mathcal{Y}_1^m(\frac{\mathbf{p}_3-\mathbf{p}_4}{2})\chi^{34}_{1-m}\phi_0^{34}\omega_0^{34} b_3^\dagger(\mathbf{p}_3) d_4^\dagger(\mathbf{p}_4), \qquad (1)$$

where γ, which is a dimensionless parameter, represents the probability of the quark-antiquark pair created from the vacuum and can be extracted by fitting observed experimental data. $\phi_0^{34} = (u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$, $\omega_0^{34} = (R\bar{R}+G\bar{G}+B\bar{B})/\sqrt{3}$ are flavor and color singlet state, respectively. $\chi^{34}_{1,-m}$ is a spin-triplet state. $\mathcal{Y}_l^m(\mathbf{p}) \equiv |p|^l Y_l^m(\theta_p,\phi_p)$ is the lth solid harmonic polynomial that reflects the momentum-space distribution of the created quark-antiquark pair. $b_3^\dagger(\mathbf{p}_3)$, $d_4^\dagger(\mathbf{p}_4)$ are the creation operators of the quark and antiquark, respectively.

In general, the mock state is adopted to describe the meson with the spatial wave function $\psi_{n_A L_A M_{L_A}}(\mathbf{p}_1,\mathbf{p}_2)$ in the momentum representation [33].

$$|A(n_A{}^{2S_A+1}L_{A\ J_A M_{J_A}})(\mathbf{P}_A)\rangle \equiv \sqrt{2E_A}\sum_{M_{L_A},M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle \times \int d\mathbf{p}_A \psi_{n_A L_A M_{L_A}}(\mathbf{p}_1,\mathbf{p}_2)\chi^{12}_{S_A M_{S_A}}\phi_A^{12}\omega_A^{12}|q_1(\mathbf{p}_1)\bar{q}_2(\mathbf{p}_2)\rangle, \qquad (2)$$

with the normalization conditions

$$\langle A(n_A{}^{2S_A+1}L_{A\ J_A M_{J_A}})(\mathbf{P}_A)|A(n_A{}^{2S_A+1}L_{A\ J_A M_{J_A}})(\mathbf{P'}_A)\rangle = 2E_A\delta^3(\mathbf{P}_A-\mathbf{P'}_A). \qquad (3)$$

where n_A represent the radial quantum number of the meson A composed of q_1, $\bar{q}_2$ with momentum $\mathbf{p_1}$ and $\mathbf{p_2}$. E_A is the total energy, $\mathbf{P}_A$ is the momentum of the meson A and $\mathbf{p}_A = (m_1\mathbf{p}_1 - m_1\mathbf{p}_2)/(m_1+m_2)$ is the relative momentum between quark and antiquark. $\mathbf{S}_A = \mathbf{s}_{q_1}+\mathbf{s}_{q_2}$, $\mathbf{J}_A = \mathbf{L}_A + \mathbf{S}_A$ stand for the total spin and total angular momentum, respectively. $\mathbf{L}_A$ is the relative orbital angular momentum between q_1 and $\bar{q}_2$. $\langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle$ denotes a Clebsch-Gordan coefficient, and $\chi^{12}_{S_A M_{S_A}}$, ϕ_A^{12} and ω_A^{12} are the spin, flavor and color wave functions, respectively.

The S-matrix of the process $A \to B + C$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi i\delta(E_A - E_B - E_C)\langle BC|T|A\rangle, \qquad (4)$$

with

$$\langle BC|T|A\rangle = \delta^3(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \quad (5)$$

where $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ is the helicity amplitude of $A \to B + C$. In the center of mass frame of meson A, $\mathbf{P}_A = 0$, and $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ can be written as

$$\begin{aligned}
\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P}) \;=\; & \gamma\sqrt{8E_AE_BE_C} \sum_{\substack{M_{L_A},M_{S_A},\\ M_{L_B},M_{S_B},\\ M_{L_C},M_{S_C},m}} \langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle \\
& \times\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle\langle 1m1-m|00\rangle\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle \\
& \times[\langle\phi^{14}_B\phi^{32}_C|\phi^{12}_A\phi^{34}_0\rangle\mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P},m_1,m_2,m_3) \\
& +(-1)^{1+S_A+S_B+S_C}\langle\phi^{32}_B\phi^{14}_C|\phi^{12}_A\phi^{34}_0\rangle\mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(-\mathbf{P},m_2,m_1,m_3)], \qquad (6)
\end{aligned}$$

with the momentum space integral,

$$\mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P},m_1,m_2,m_3) = \int d\mathbf{p}\,\psi^*_{n_BL_BM_{L_B}}(\tfrac{m_3}{m_1+m_3}\mathbf{P}+\mathbf{p})\psi^*_{n_CL_CM_{L_C}}(\tfrac{m_3}{m_2+m_3}\mathbf{P}+\mathbf{p})\psi_{n_AL_AM_{L_A}}(\mathbf{P}+\mathbf{p})\mathcal{Y}^m_1(\mathbf{p}), \quad (7)$$

where $\mathbf{P} = \mathbf{P}_B = -\mathbf{P}_C$, $\mathbf{p} = \mathbf{p}_3$, m_3 is the mass of the created quark q_3; $\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle$ and $\langle\phi^{14}_B\phi^{32}_C|\phi^{12}_A\phi^{34}_0\rangle$ are the overlap of spin and flavor wave function, respectively.

The spin overlap in terms of Winger's $9j$ symbol can be given by

$$\begin{aligned}
&\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle = \\
&\sum_{S,M_S}\langle S_BM_{S_B}S_CM_{S_C}|SM_S\rangle\langle S_AM_{S_A}1-m|SM_S\rangle \\
&\times(-1)^{S_C+1}\sqrt{3(2S_A+1)(2S_B+1)(2S_C+1)} \\
&\times\left\{\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & S_A \\ \frac{1}{2} & \frac{1}{2} & 1 \\ S_B & S_C & S \end{array}\right\}. \qquad (8)
\end{aligned}$$

Generally, one takes the simple harmonic oscillator (SHO) approximation for the meson space wave functions in Eq. (7). In momentum-space, the SHO wave function reads

$$\begin{aligned}
\Psi_{nLM_L}(\mathbf{p}) &= (-1)^n(-i)^LR^{L+\frac{3}{2}}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}} \\
&\times\exp\left(-\frac{R^2p^2}{2}\right)L_n^{L+\frac{1}{2}}\left(R^2p^2\right)\mathcal{Y}_{LM_L}(\mathbf{p}), \qquad (9)
\end{aligned}$$

with $\mathcal{Y}_{LM_L}(\mathbf{p}) = |\mathbf{p}|^LY_{LM_L}(\Omega_p)$. Here R denotes the SHO wave function scale parameter; $\mathbf{p}$ represents the relative momentum between the quark and the antiquark within a meson; $L_n^{L+\frac{1}{2}}\left(R^2p^2\right)$ is an associated Laguerre polynomial.

The decay width for the process $A \to B + C$ in terms of the helicity amplitude is

$$\Gamma = \pi^2\frac{|\mathbf{P}|^2}{M_A^2}\frac{1}{2J_A+1}\sum_{\substack{M_{J_{M_A}},M_{J_{M_B}},\\ M_{J_{M_C}}}}\left|\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}\right|^2.$$

For comparing with experiments, $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P})$ can be converted into the partial amplitude via the Jacob-Wick formula [34]

$$\begin{aligned}
\mathcal{M}^{JL}(A\to BC) &= \frac{\sqrt{2L+1}}{2J_A+1}\sum_{M_{J_B},M_{J_C}}\langle L0JM_{J_A}|J_AM_{J_A}\rangle \\
&\times\langle J_BM_{J_B}J_CM_{J_C}|JM_{J_A}\rangle\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P}), \qquad (10)
\end{aligned}$$

where $\mathbf{J} = \mathbf{J}_B + \mathbf{J}_C$, $\mathbf{J}_A = \mathbf{J}_B + \mathbf{J}_C + \mathbf{L}$,and $M_{J_A} = M_{J_B} + M_{J_C}$. Then the decay width in terms of the partial wave amplitude is taken as,

$$\Gamma = \pi^2\frac{|\mathbf{P}|}{M_A^2}\sum_{JL}\left|\mathcal{M}^{JL}\right|^2, \qquad (11)$$

where $|\mathbf{P}|$, as mentioned above, is the three momentum of the outgoing meson in the rest frame of meson A. According to the calculation of 2-body phase space, one can get

$$|\mathbf{P}| = \frac{\sqrt{[M_A^2-(M_B+M_C)^2][M_A^2-(M_B-M_C)^2]}}{2M_A},$$

where M_A, M_B, and M_C are the masses of the meson A, B, and C, respectively.

III. THE POSSIBLE STRONG DECAY CHANNELS AND AMPLITUDES OF THE CANDIDATES FOR THE $X(4160)$ AND $X(3915)$

As analyzed in section I, we consider the $\eta_c(4^1S_0)$, $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$ as the possible candidates of the $X(4160)$, and assume that the upper limit of the mass is 4156 MeV observed by Belle. For the X(3915), one chooses charmonium state $\chi_0(2^3P_0)$ with mass 3916 MeV. According to the $^3P_0$ model discussed in the above section, the OZI rule allows open-charm strong decay and corresponding amplitudes of possible charmonium states are listed in Tables II and III. We replace $\mathcal{I}_{0,0}^{+1-1}, \mathcal{I}_{0,0}^{-1+1}$ with $\mathcal{I}^{\pm}$ and $\mathcal{I}_{0,0}^{0,0}$ with $\mathcal{I}^{0,0}$ in Table III, respectively. The details of the spatial integral about $\mathcal{I}^{\pm}(\mathbf{P})$ and $\mathcal{I}^{0,0}(\mathbf{P})$ are given in the Appendix.

TABLE II: The OZI rule and phase space allowed open-charm strong decay modes of the possible charmonium states for the X(4160) and X(3915).

State	J^{PC}	Decay mode	Decay channel
$\eta_c(4^1S_0)$	0^{-+}	0^-+1^-	$D\bar{D}^*, D_s^+D_s^{*-}$
		1^-+1^-	$D^*\bar{D}^*$
$\chi_0(3^3P_0)$	0^{++}	0^-+0^-	$D\bar{D}, D_s^+D_s^-$
		1^-+1^-	$D^*\bar{D}^*$
$\chi_1(3^3P_1)$	1^{++}	0^-+1^-	$D\bar{D}^*, D_s^+D_s^{*-}$
		1^-+1^-	$D^*\bar{D}^*$
$\eta_{c2}(2^1D_2)$	2^{-+}	0^-+1^-	$D\bar{D}^*, D_s^+D_s^{*-}$
		1^-+1^-	$D^*\bar{D}^*$
$\chi_0(2^3P_0)$	0^{++}	0^-+0^-	$D\bar{D}$

TABLE III: The partial wave amplitude for the strong decays of relevant charmonium state. The element of flavor matrix $\langle\phi_B^{14}\phi_C^{32}|\phi_A^{12}\phi_0^{34}\rangle = 1/\sqrt{3}$ in present work. We take $\mathcal{E} = \gamma\sqrt{E_AE_BE_C}$ in this table.

State	decay channel	Decay amplitude
$\eta_c(4^1S_0)$	0^-+1^-	$\mathcal{M}^{11} = \frac{\sqrt{2}}{3}\mathcal{E}\mathcal{I}^{00}$
	1^-+1^-	$\mathcal{M}^{11} = \frac{2}{3}\mathcal{E}\mathcal{I}^{00}$
$\chi_0(3^3P_0)$	0^-+0^-	$\mathcal{M}^{00} = \frac{\sqrt{2}}{3\sqrt{3}}\mathcal{E}\left(\mathcal{I}^{00}-2\mathcal{I}^{\pm}\right)$
	1^-+1^-	$\mathcal{M}^{00} = \frac{\sqrt{2}}{9}\mathcal{E}\left(\mathcal{I}^{00}-2\mathcal{I}^{\pm}\right)$
		$\mathcal{M}^{22} = \frac{4}{9}\mathcal{E}\left(\mathcal{I}^{00}+\mathcal{I}^{\pm}\right)$
$\chi_1(3^3P_1)$	0^-+1^-	$\mathcal{M}^{10} = \frac{2}{9}\mathcal{E}\left(\mathcal{I}^{00}-2\mathcal{I}^{\pm}\right)$
		$\mathcal{M}^{12} = \frac{\sqrt{2}}{9}\mathcal{E}\left(\mathcal{I}^{00}+\mathcal{I}^{\pm}\right)$
	1^-+1^-	$\mathcal{M}^{22} = \frac{2}{3\sqrt{3}}\mathcal{E}\left(\mathcal{I}^{00}+\mathcal{I}^{\pm}\right)$
$\eta_{c2}(2^1D_2)$	0^-+1^-	$\mathcal{M}^{11} = \frac{2}{15}\mathcal{E}\left(\sqrt{3}\mathcal{I}^{\pm}-\mathcal{I}^{00}\right)$
	1^-+1^-	$\mathcal{M}^{11} = \frac{2\sqrt{2}}{15}\mathcal{E}\left(\sqrt{3}\mathcal{I}^{\pm}-\mathcal{I}^{00}\right)$
$\chi_0(2^3P_0)$	0^-+0^-	$\mathcal{M}^{00} = \frac{\sqrt{2}}{3\sqrt{3}}\mathcal{E}\left(\mathcal{I}^{00}-2\mathcal{I}^{\pm}\right)$

IV. NUMERICAL RESULTS AND DISCUSSION

There are several parameters should be input to calculate the strong decay in the $^3P_0$ model. In the present work, the masses of constituent quarks are taken as $m_u = m_d = 0.22$ GeV, $m_s = 0.419$ GeV, $m_c = 1.6$ GeV [36]. The strength of quark pair creation $\gamma = 6.95$ has been adopted by many literatures [22, 27], which is fitted by strong decay of light-, charmonium-, open charmed-mesons and baryons observed by experiments. The value of γ is higher than that used in Ref. [37] by a factor of $\sqrt{96\pi}$ due to different field theory conventions. The strength of $s\bar{s}$ creation satisfies $\gamma_s = \gamma/\sqrt{3}$ [38]. Refs.[21, 22, 23] also take this value to study the strong decay of charmonium, heavy-light meson and heavy baryons. In this work, we take these parameters for calculation as well. The R values of D, D^*, D_s, D_s^* in the SHO are shown in Table IV, which are obtained by the calculation of the nonrelativistic quark model with Coulomb item, linear confinement and smeared hyperfine interactions.

TABLE IV: The parameters relevant to the two-body strong decays of the charmonium state in the $^3P_0$ model.

State	Mass (MeV) [35]	R (GeV^{-1}) [36]
D	1869.62(±) 1864.84(0)	1.52
D^*	2021.27(±) 2006.97(0)	1.85
D_s	1968.49(±)	1.41
D_s^*	2112.3(±)	1.69

First of all, we study the strong decay of the $\chi_0(3^3P_0)$ which is discussed by Chao and Li in Refs.[6, 12] from the production process of $e^+e^- \to J/\psi + X(4160)$ and the mass spectrum is obtained by the potential model with color screening. Using the method of Numerov algorithm [39], we also obtain the mass 4149 MeV by the same potential and parameters in Ref. [6]. Usually, the width of strong decay is sensitive [20, 21, 22, 24, 27, 32] to the R value in the SHO. Here the reasonable value of R is obtained by fitting the wave function obtained by solving the schrödinger equation [6].

Through the Fourier transform, the Eq. (9) turns into

$$\Psi_{nLM_L}(\mathbf{r}) = R_{nL}(r)Y_{LM_L}(\Omega_r), \tag{12}$$

with the radial wave function

$$R_{nL}(r) = R^{-(L+\frac{3}{2})}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}} \times \exp\left(-\frac{R^{-2}r^2}{2}\right) r^L L_n^{L+\frac{1}{2}}\left(R^{-2}r^2\right). \tag{13}$$

The wave function $u(r) = r\,R_{nL}(r)$ of charmonium state $3P$ is shown in Fig.2. Using Eq.(13) to fit the wave function got by Numerov algorithm method (the wave function is denoted as 'NAWF' in the following), we can get the $R = 2.5 \sim 2.98$ GeV^{-1}.

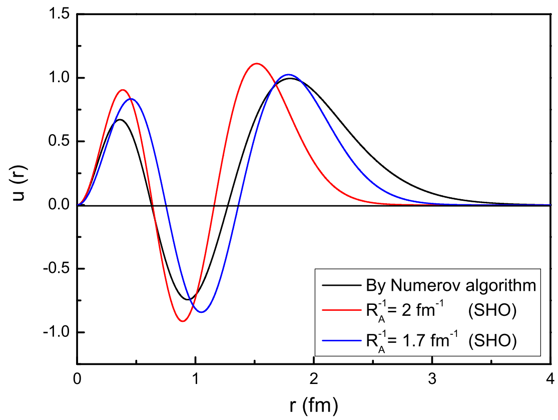

Fig.2 The wave function of charmonium state $3P$.

The $\chi_0(3^3P_0)$ has decay channels of $0^{++} \to 0^- + 0^-$ with S-wave and $0^{++} \to 1^- + 1^-$ with S-, D-wave, while the $0^{++} \to 0^- + 1^-$ is forbidden. Therefore, it can decay into $D\bar{D}$, D_sD_s, $D^*\bar{D}^*$, which are allowed by the phase space. In Fig.3, we show the dependence of the partial widths of the strong decay of the $\chi_0(3^3P_0)$ on the R_A. Taking $R_A = 2.5 \sim 2.98$ GeV^{-1} discussed above, the total width ranges from 105 to 143 MeV which falls in the range of experimental data. However, the dominate contribution comes from the $\chi_0(3^3P_0) \to DD$ which is inconsistent with the experimental result. So the assignment of the charmonium state $\chi_0(3^3P_0)$ to the $X(4160)$ is disfavored.

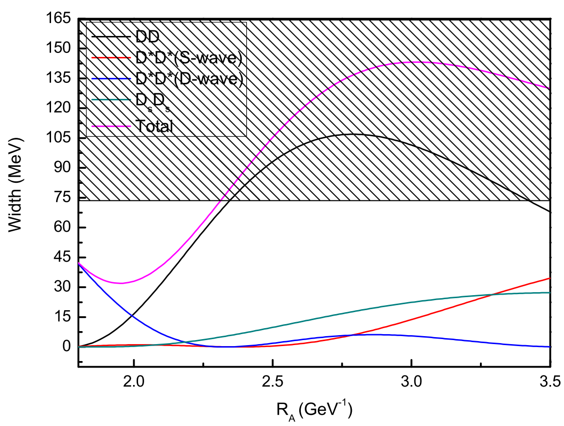

Fig.3 The possible strong decay of the $\chi_0(3^3P_0)$.

The $\eta_c(4^1S_0)$ is mostly like the $X(4160)$ for it has high production cross sections in the process of $e^+e^- \to J/\psi + X(4160)$ discussed by Chao [12]. However, it is difficult to understand why the predicted mass 4250 MeV [6], 4384, 4425 MeV [13] are much higher than 4156 MeV. By considering the effect of the meson loops [40], the mass may be lower than that of Refs.[6, 13]. Here, we assume the mass of the $\eta_c(4^1S_0)$ is 4156 MeV. The main decay channels of the $\eta_c(4^1S_0)$ are $0^{-+} \to 0^- + 1^-$ and $0^{-+} \to 1^- + 1^-$ with P-wave between outgoing mesons. Obviously, the $0^{-+} \to 0^- + 0^-$ is forbidden. The decay width of main decay channels are shown in Fig.4. The total width can only reach up to about 25 MeV with R_A around 2.9 GeV, which is obtained by fitting to NAWF of the $\eta_c(4^1S_0)$. It is about 3 times smaller than the lower limit of the experimental result of the $X(4160)$. Since the results of some hadron states predicted by the $^3P_0$ model may be a factor of $2 \sim 3$ off the experimental width due to inherent uncertainties of this model [16, 17, 18, 19, 27], the assignment of the X(4160) to the $\eta_c(4^1S_0)$ cannot be excluded. The ratio of main decay channel $D\bar{D}^*$, $D^*\bar{D}^*$ is

$$\frac{\mathcal{B}(\eta_c(4^1S_0) \to D\bar{D}^*)}{\mathcal{B}(\eta_c(4^1S_0) \to D^*\bar{D}^*)} = 1.25. \tag{14}$$

It is much larger than the 0.22 reported by Belle. If one takes the $\eta_c(4^1S_0)$ as an assignment of X(4160), the precision measurement of the ratio between the width of the $D\bar{D}^*$ and $D^*\bar{D}^*$ is necessary in further experiment.

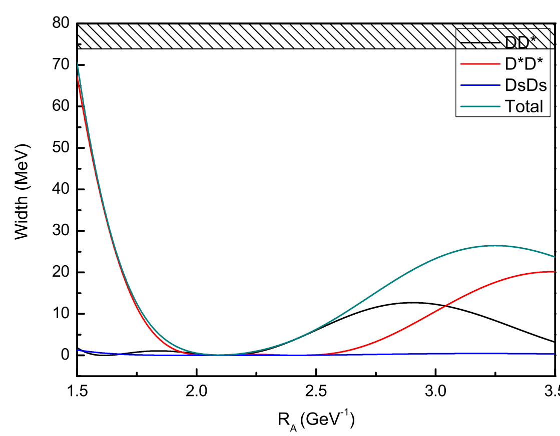

Fig.4 The possible strong decay of the $\eta_c(4^1S_0)$.

Because the $\chi_1(3^3P_1)$ has quantum number $J^{PC} = 1^{++}$ and mass 4178 MeV, it is also a possible candidate of the $X(4160)$. $1^{++} \to 0^- + 1^-$ and $1^{++} \to 1^- + 1^-$ with S- and D-wave are the main decay channels of the $\chi_1(3^3P_1)$. Fig.5 shows our results in the $^3P_0$ model. Taking $R_A = 2.5 \sim 2.98$ GeV^{-1}, the total width is consistent with the range of the $X(4160)$. However, the dominant decay is $\chi_1(3^3P_1) \to D\bar{D}^*$ while the decay width has only a few MeV for the $\chi_1(3^3P_1) \to D^*\bar{D}^*$ channel, which is inconsistent with the experimental data. Therefore, regarding the $X(4160)$ as the $\chi_1(3^3P_1)$ state is impossible.

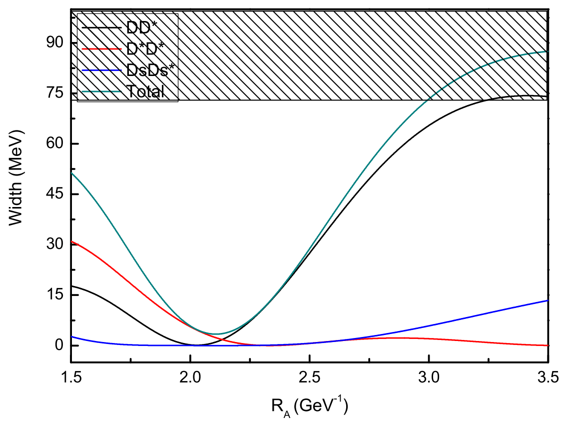

Fig.5 The possible strong decay of the $\chi_1(3^3P_1)$.

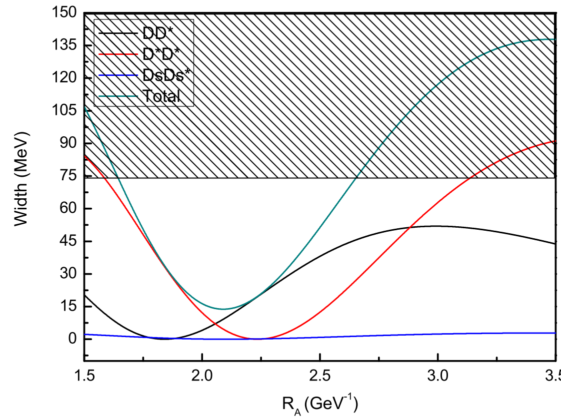

Fig.6 The possible strong decay of the $\eta_{c2}(2^1D_2)$.

The another possible candidate of the $X(4160)$ is the charmonium state $\eta_{c2}(2^1D_2)$. Firstly, it has quantum number $J^{PC}=2^{-+}$ and mass 4099 MeV [6], 4158 MeV [13] which are compatible with the result of Belle. Secondly, the $\psi(4160)$[35] is known to be the good candidate of the $\psi(2^3D_1)$ with $J^{PC}=1^{--}$, which is discussed in detail by Chao [12]. So the $X(4160)$ may be the D-wave spin-singlet charmonium state $^1D_2(2D)$. Thirdly, $\eta_{c2}(2^1D_2)$ decaying into $D\bar{D}$ is forbidden, and this decay is also not seen by Belle.

For the strong decay of the $\eta_{c2}(2^1D_2)$, it has $2^{-+}\to 0^-+1^-$ and $2^{-+}\to 1^-+1^-$ decay channels with P-wave between outgoing mesons. In this case, final states $D\bar{D}^*$, $D_s\bar{D}_s^*$ and $D^*\bar{D}^*$ are phase space allowed. In Fig.6, we present the numerical results of main decay channels for the $\eta_{c2}(2^1D_2)$. By fitting the NAWF of the $\eta_{c2}(2^1D_2)$, we get $R_A=2.7\sim 3.0$ GeV^{-1}. The total decay width of the $\eta_{c2}(2^1D_2)$ falls in the range of the $X(4160)$ released by Belle. Taking the reasonable R_A value of the SHO, the ratio of the main decay channel $D\bar{D}^*$, $D^*\bar{D}^*$ is

$$\frac{\mathcal{B}(\eta_{c2}(2^1D_2)\to D\bar{D}^*)}{\mathcal{B}(\eta_{c2}(2^1D_2))\to D^*\bar{D}^*)}=1.4\sim 0.76 \qquad (15)$$

and shown in Fig.7. However, the result is somewhat larger than the $\mathcal{B}_{D^*\bar{D}}(X(4160))/\mathcal{B}_{D^*\bar{D}^*}(X(4160))<0.22$ observed by Belle. We believe that to measure this ratio is very important since it is independent on the uncertain strength γ of the quark pair creation from vacuum.

To sum up, the $\eta_{c2}(2^1D_2)$ is a better candidate for the $X(4160)$ in the present calculation.

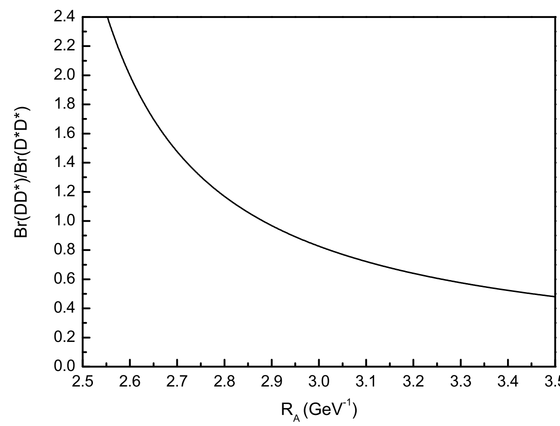

Fig.7 The ration of $\frac{\mathcal{B}(\eta_{c2}(2^1D_2)\to D\bar{D}^*)}{\mathcal{B}(\eta_{c2}(2^1D_2))\to D^*\bar{D}^*)}$ with R_A value of the SHO.

The $X(3915)$, which was observed by Belle in $\gamma\gamma\to\omega J/\psi$ with a statical significance of 7.5σ [8], is the most recent addition to the collection of the XYZ states. According to the Table I predicted by potential model, the excited charmonium state $\chi_0(2^3P_0)$ is a good candidate for the $X(3915)$, due to it has mass $M=3914\pm 4\pm 2$ MeV and the possible quantum number is $J^{PC}=0^{++}$.

The $\chi_0(2^3P_0)$ has only the strong decay channel $0^{++}\to 0^-+0^-$ allowed by phase space. The width of $\chi_0(2^3P_0)\to D\bar{D}$ with R_A of the SHO is presented in Fig.8. The total width ranges from 132 to 187 MeV with $R_A=2.3\sim 2.5$ GeV^{-1} fitted to the NAWF of the $\chi_0(2^3P_0)$. It is much larger than the $\Gamma=28\pm 12^{+2}_{-8}$ MeV reported by Refs. [8, 9, 10]. Therefore, the $X(3915)$ is unlikely to be the charmonium state $\chi_0(2^3P_0)$ although the mass is compatible with the $X(3915)$.

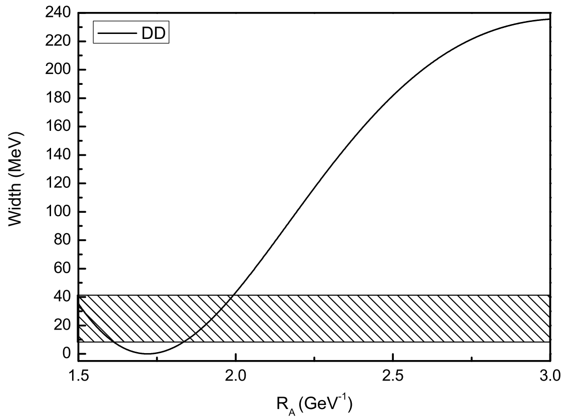

Fig.8 The possible strong decay of the $\chi_0(2^3P_0)$.

V. SUMMARY

In summary, we have discussed the possible interpretations of the $X(4160)$ observed by Belle collaborations in $e^+e^- \to J/\psi + X(4160)$ followed by $X(4160) \to D^*\bar{D}^*$. We also study the newest state $X(3915)$ observed by Belle in the process $\gamma\gamma \to J/\psi\omega$ [8].

In quark models, the masses of the charmonium states: $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$ are all around 4156 MeV. By taking the effect of virtual mesons loop [40] into account, the $\eta_c(4^1S_0)$ may also has mass around 4156 MeV. All the four states have charge parity $C = +$ which are compatible with the $X(4160)$ observed by Belle.

For the strong decay of the $\chi_0(3^3P_0)$, the dominant strong decay is $\chi_0(3^3P_0) \to D\bar{D}$ while $\chi_0(3^3P_0) \to D^*\bar{D}^*$ contributes to the total width only a little in the reasonable R in the SHO. It is contrast to the experimental result. Thus the excited charmonium state $\chi_0(3^3P_0)$ disfavor the $X(4160)$.

The $\eta_c(4^1S_0)$ can not decay into $D\bar{D}$ and may has high production rate [12] in $e^+e^- \to J/\psi + \eta_c(4S)$ process by analogy with $e^+e^- \to J/\psi + \eta_c(1S)(\eta_c(2S)\chi_{c0}(1P))$. However, the total width in present work is lower than the experimental data of the $X(4160)$.

The main strong decay channel of the $\chi_1(3^3P_1)$ is $D\bar{D}^*$ while $D^*\bar{D}^*$ is only a few MeV. It is inconsistent with the results of Belle. Therefore, taking the $\chi_1(3^3P_1)$ as an assignment for the $X(4160)$ is impossible.

The $\eta_{c2}(2^1D_2)$ can not decay to $D\bar{D}$ which is also not seen in the experiment. The total width of the $\eta_{c2}(2^1D_2)$ match well with the data of the $X(4160)$ in our calculation. So, the $\eta_{c2}(2^1D_2)$ is a good candidate for the $X(4160)$, for it is not only the mass but also the strong decay are well compatible with the results observed by Belle, although the excited charmonium state $\eta_c(4^1S_0)$ can not be rule out as an assignment for the $X(4160)$.

We also give the ratio of $\frac{\mathcal{B}(\eta_{c2}(2^1D_2)\to D\bar{D}^*)}{\mathcal{B}(\eta_{c2}(2^1D_2))\to D^*\bar{D}^*)}$ which is independent on the parameter γ in the $^3P_0$ model. The numerical result is somewhat larger than the experimental data. Therefore, we suggest Belle, BaBar and other experimental collaborations to measure it to confirm this state.

By assuming the $X(3915)$ is the $\chi_0(2^3P_0)$, the strong decay of the state is calculated. From our numerical results, we think this assumption is unacceptable. Due to the partial width of the $X(3915)$ to $\gamma\gamma$ or $\omega J/\psi$ is too large, Yuan [9] also believes that it is very unlikely to be a charmonium state. Thus, It is necessary to do more study to understand the properties of the $X(3915)$.

Acknowledgments

You-chang Yang would like to thank Xin Liu for useful discussion. The work is supported partly by the National Science Foundation of China under Contract No.10775072 and the Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20070319007, No. 1243211601028.

Appendix

The spatial overlap $\mathcal{I}^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P}, m_1, m_2, m_3)$ is simplified as $\mathcal{I}^{n'm'}(\mathbf{P})$ in present work due to $M_{L_B} = M_{L_C} = 0$. According to the Eq. (7), the concrete calculations of the integration are trivial after choosing the direction of $\mathbf{P}$ along z axis [34]. We list all expressions of $I^{\pm}, I^{00}$ used in Table III

In the case of $2P \to 1S + 1S$

$$
\begin{aligned}
I^{\pm} &= I^{1-1} = I^{-11} \\
&= i\frac{\sqrt{6}}{\sqrt{5}\pi^{5/4}\Delta^7}\left(R_A^{5/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)(10\ R_A^2 + \Delta^2(-5 + 2\ \mathbf{P}^2R_A^2(1+\lambda)^2)) \\
I^{00} &= -i\frac{\sqrt{6}}{\sqrt{5}\pi^{5/4}\Delta^7}\left(R_A^{5/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right) \\
&\quad (10R_A^2 + \Delta^2(-5 + \mathbf{P}^2(1+\lambda)(-5\ \Delta^2\lambda + 2\ R_A^2(3 + \lambda(8 + \Delta^2\mathbf{P}^2(1+\lambda)^2)))))).
\end{aligned}
\tag{16}
$$

For $2D \to 1S + 1S$

$$\begin{aligned}
I^{\pm} &= I^{1-1} = I^{-11} \\
&= \frac{2\sqrt{3}}{\sqrt{7}\pi^{5/4}\Delta^7}\left(R_A^{7/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\mathbf{P}(1+\lambda)(14\ R_A^2+\Delta^2(-7+2\ \mathbf{P}^2R_A^2(1+\lambda)^2)) \\
I^{00} &= -\frac{2}{\sqrt{7}\pi^{5/4}\Delta^7}\left(R_A^{7/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\mathbf{P}(1+\lambda)(28\ R_A^2+\Delta^2(-14+\mathbf{P}^2(1+\lambda) \\
&\quad (-7\ \Delta^2\lambda+2\ R_A^2(4+\lambda(11+\Delta^2\mathbf{P}^2(1+\lambda)^2))))). \qquad (17)
\end{aligned}$$

For $3P \to 1S + 1S$

$$\begin{aligned}
I^{\pm} &= I^{1-1} = I^{-11} \\
&= i\frac{\sqrt{3}}{\sqrt{70}\pi^{5/4}\Delta^9}\left(R_A^{5/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)(140\ R_A^4+28\ \Delta^2R_A^2(-5+2\ \mathbf{P}^2R_A^2(1+\lambda)^2) \\
&\quad +\Delta^4(35-28\ \mathbf{P}^2R_A^2(1+\lambda)^2+4\ \mathbf{P}^4R_A^4(1+\lambda)^4)) \\
I^{00} &= -i\frac{2\sqrt{6}}{\sqrt{35}\pi^{5/4}\Delta^9}\left(R_A^{5/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)(35\ R_A^4+\frac{1}{4}\ \Delta^6\mathbf{P}^2\lambda(1+\lambda) \\
&\quad (35-28\ \mathbf{P}^2R_A^2(1+\lambda)^2+4\ \mathbf{P}^4R_A^4(1+\lambda)^4)+7\ \Delta^2R_A^2(-5+\mathbf{P}^2R_A^2(1+\lambda)(6+11\ \lambda)) \\
&\quad +\frac{1}{4}\ \Delta^4(35-28\ \mathbf{P}^2R_A^2(1+\lambda)(3+8\ \lambda)+4\ \mathbf{P}^4R_A^4(1+\lambda)^3(5+19\ \lambda))). \qquad (18)
\end{aligned}$$

For $4S \to 1S + 1S$

$$\begin{aligned}
I^{00} &= \frac{1}{2\sqrt{120}\pi^{5/4}\Delta^9}\left(R_A^{3/2}R_B^{3/2}R_C^{3/2}\right)\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\mathbf{P}(840\ R_A^6(2+3\ \lambda)+\Delta^6\lambda(-105+210\ \mathbf{P}^2R_A^2(1+\lambda)^2 \\
&\quad -84\ \mathbf{P}^4R_A^4(1+\lambda)^4+8\ \mathbf{P}^6R_A^6(1+\lambda)^6)+6\ \Delta^4R_A^2(70+175\ \lambda-28\ \mathbf{P}^2R_A^2(1+\lambda)^2(2+7\ \lambda) \\
&\quad +4\ \mathbf{P}^4R_A^4(1+\lambda)^4(2+9\ \lambda))+84\ \Delta^2R_A^4(-5(4+7\ \lambda)+2\ \mathbf{P}^2R_A^2(1+\lambda)^2(4+9\ \lambda))). \qquad (19)
\end{aligned}$$

Here, The parameters Δ, ζ and η in Eqs. (16), (17), (18), (19) are defined as

$$\Delta^2 = R_A^2+R_B^2+R_C^2,\ \lambda = -\frac{R_A^2+\xi_1R_B^2+\xi_2R_C^2}{R_A^2+R_B^2+R_C^2},$$
$$\zeta^2 = R_A^2+\xi_1^2R_B^2+\xi_2^2R_C^2-\frac{(R_A^2+\xi_1R_B^2+\xi_2R_C^2)^2}{R_A^2+R_B^2+R_C^2}.$$

with

$$\xi_1 = \frac{m_3}{m_3+m_1},\quad \xi_2 = \frac{m_3}{m_3+m_2}.$$

Here m_1, m_2 and m_3 denotes the mass of quark inside parent meson and created from vacuum, respectively.

[1] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **17**, 3090 (1978).

[2] Estia J. Eichten, Kenneth Lane, and Chris Quigg, Phys. Rev. D **73**, 014014 (2006).

[3] T. Barnes and E. S. Swanson, Phys. Rev. C **77**, 055206 (2008)

[4] Bai-Qing Li, Ce Meng, and Kuang-Ta Chao, Phys. Rev. D **80**, 014012 (2009)

[5] M. Suzuki, Phys. Rev. D **72**, 114013 (2005).

[6] Bai-Qing Li and Kuang-Ta Chao, Phys. Rev. D **79**, 094004 (2009).

[7] P. Pakhlov *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 202001 (2008), arXiv: 0708.3812 [hep-ex].

[8] S. L. Olsen, arXiv: 0909.2713 [hep-ex].

[9] C. Z. Yuan [BES and Belle collaborations], arXiv: 0910.3138 [hep-ex].

[10] A. Zupanc [Belle collaboration], arXiv: 0910.3404 [hep-ex].

[11] Stephen Gordfrey, arXiv: 0910.3409 [hep-ph].

[12] K. T. Chao, Phys. Lett. B **661**, 348 (2008).

[13] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[14] Kui-Yong Liu, Zhi-Guo He, and Kuang-Ta Chao, Phys. Rev. D **77**, 014002 (2008).
[15] R. Molina and E. Oset, arXiv: 0907.3043 [hep-ph].
[16] L. Micu, Nucl. Phys. B **10**, 521 (1969).
[17] A. Le Yaouanc, L. Oliver, O. Pene, J-C. Raynal, Phys. Rev. D **8**, 2223 (1973); Phys. Rev. D **9**, 1415 (1974); Phys. Rev. D **11**, 1272 (1975).
[18] W. Roberts and B. Silvestr-Brac, Few-Body Syst. **11**, 171 (1992).
[19] E. S. Ackleh, T. Barnes and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).
[20] T. Barnes, S. Godfrey, E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[21] Xiang Liu, Zhi-Gang Luo, and Zhi-Feng Sun, arXiv: 0911. 3694 [hep-ph].
[22] J. Lu, W. Z. Deng, X. L. Chen, S. L. Zhu, Phys. Rev. D **73**, 054012 (2006); B. Zhang, X. Liu, W. Z. Deng, S. L. Zhu, Eur. Phys. J. C **50**, 617 (2007); C. Chen, X. L. Chen, X. Liu, W. Z. Deng, S. L. Zhu, Phys. Rev. D **75**, 094017 (2007).
[23] Zhi-Gang Luo, Xiao-Lin Chen, Xiang Liu, Phys. Rev. D **79**, 074020 (2009); Zhi-Feng Sun and Xiang Liu, Phys. Rev. D **80**, 074037 (2009).
[24] F. E. Close, E. S. Swanson, Phys. Rev. D **72**, 094004 (2005); F. E. Close, C. E. Thomas, O. Lakhina, E. S. Swanson, Phys. Lett. B **647**, 159 (2007); O. Lakhina, E. S. Swanson, Phys. Lett. B **650**, 159 (2007).
[25] S. Capstick, N. Isgur, Phys. Rev. D **34**, 2809 (1986); S. Capstick, W. Roberts, Phys. Rev. D **49** 4570 (1994).
[26] P. Geiger, E. S. Swanson, Phys. Rev. D **50**, 6855 (1994)
[27] H.G. Blundell, S. Godfrey, Phys. Rev. D **53**,3700 (1996); Phys. Rev. D **53**, 3712 (1996).
[28] R. Kokoski, N. Isgur, Phys. Rev. D **35**, 907 (1987).
[29] T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997).
[30] T. Barnes, N. Black and P. R. Page, Phys. Rev. D **68**, 054014 (2003).
[31] L. Burakovsky, P. R. Page, Phys. Rev. D **62**, 014011 (2000).
[32] De-Min Li and Bing Ma, Phys. Rev. D **77**, 074004 (2008); Phys. Rev. D **77**, 094021 (2008); Phys. Rev. D **79** 014014, (2009); arXiv: 0911. 2906 [hep-ph].
[33] C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982).
[34] M. Jacob and G. C. Wick, Annals Phys. **7**, 404 (1959) [Annals Phys. **281**, 774 (2000)].
[35] C. Amsler, *et al.* Partical Data Group, Phys. Lett. B **667**, 1 (2008).
[36] S. Godfrey and R. Kokoski, Phys. Rev. D **43**, 1679 (1991).
[37] Richard. Kokoski, Nathan. Isgur, Phys. Rev. D **35**, 907 (1987).
[38] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B **72**, 57 (1977).
[39] S. E. Koonin and D. C. Meredith, *Computational Physics* (Addison-Wesley, New York, 1990).
[40] This work is in progress by our group.